Heterotic little Strings, T-duality and 2-Groups

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Based on upcoming works with M. del Zotto & M. Liu [1]

- **arXiv:2207.XXXXX**

String Phenomenology 2022
Liverpool, United Kingdom, 5th of July 2022

[1] More details in Muyangs talk on Thursday

Why 6D Little String Theories (LSTs)?

6D SUSY theories **great testing ground**:

- Understand **String Vacua/Landscape**,
- **Symmetries and non-perturbative** phenomena

6D SUGRA
Scale: Planck Scale
Global Symmetries: X
T-duality: ✓

6D QFT $\rightarrow$ SCFT
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Little Strings (LSTs) capture **features of** both **SCFTs** and **SUGRAs**:

1. LSTs have **Global symmetries** AND can be related via **T-duality**
2. Related by **decompactification**: SUGRA $\to$ LSTs $\to$ SCFTs
3. **Geometric control** via F-theory [Bhardwaj, Del Zotto, Heckman, . Morrison, Rudelius, Vafa'15]

Higher Symmetries

Higher form symmetries generalize group actions to act on p-dimensional operators C_p [Gaiotto, Kapustin, Seiberg , Willet'15]

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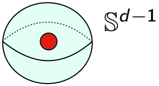

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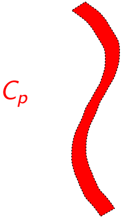

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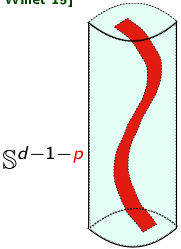

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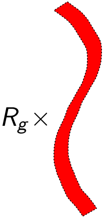

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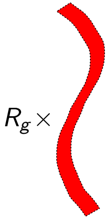

1. Goes **beyond Lagrangian** descriptions
2. **Higher form symmetries** of different form-degree **mix to Higher groups**

2-Groups in Little Strings Theories (LST)s

LSTs have **generic** continuous **2-group symmetry** [Córdova,Dumitrescu, Intriligator'18]

2-Group Symmetry Structure Constants

$$ {}^2G_{P,R,F} = \left(P^{(0)} \times SU(2)_R^{(0)} \times G_F^{(0)} \right) \times_{\hat{\kappa}_F,\hat{\kappa}_P,\hat{\kappa}_R} \mathfrak{u}_1^{(1)}, $$

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- $\eta^{I,J}$: **negative semi-definite Dirac Paring**
- $h^\vee_{G_I}$: dual coxeter number of gauge group G_I coupled to $I-$th tensor
- N_I tensor charges under little string $\mathfrak{u}_1^{(1)}$

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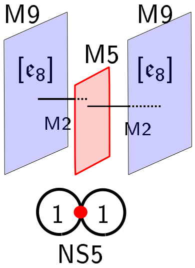
Goal: New T-dual Little strings and match 2-group Symmetries

Heterotic little strings in Horava-Witten Picture



- Place one or multiple **M5 branes** between **two M9 walls**

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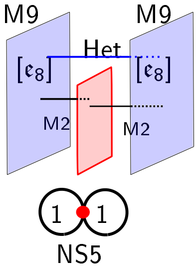
[Haghighat, Lockhart, Vafa'14]



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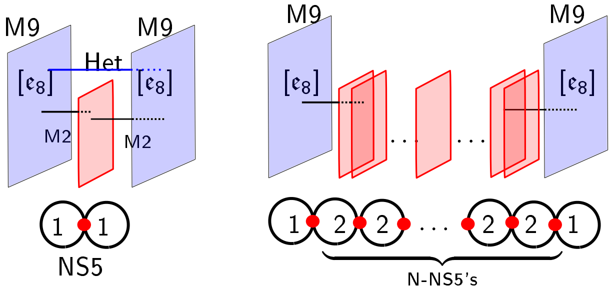

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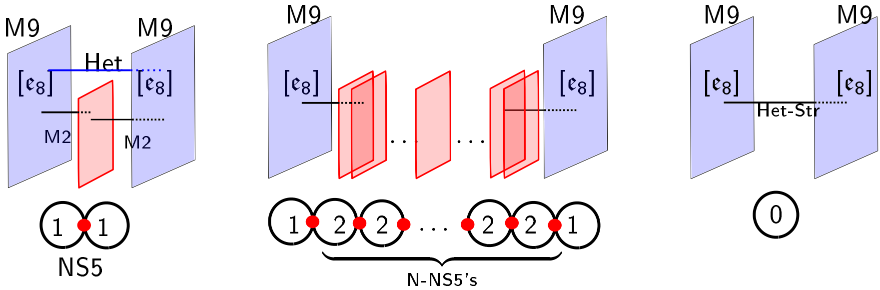

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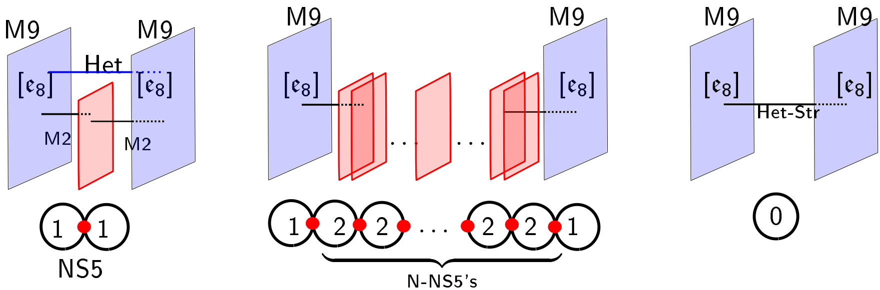

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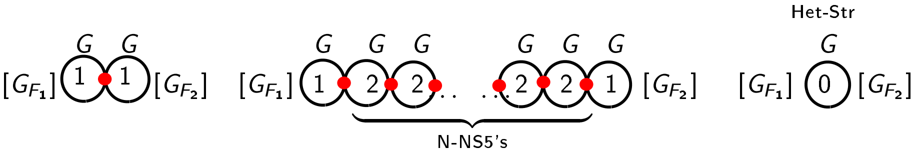
- $E_8^2 - Spin(32)/\mathbb{Z}_2$ Duality $\rightarrow$ $Sp(N)$ gauge group [Aspinwall, Morrison'96]
- 2-group structure constants match ✓ [del Zotto, Ohmori'20]

New Heterotic LSTs



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2. **Specify two flat connections** $\mu_i(E_8)$ breaking to $G_F = [\mu_i, E_8]$

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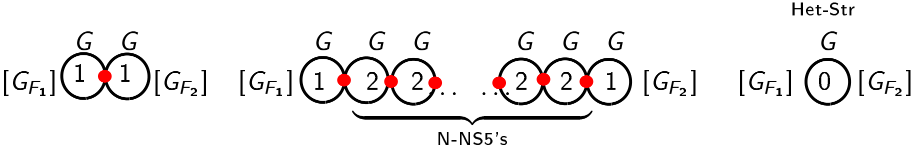

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- Leads to **fractionalization**, obtained by **fusion** of 6D SCFTs
  - M9-M5: $T(\mu_i, G)$ **orbi-instanton** theories [Kac'83, Frey'98 & Rudelius'18]
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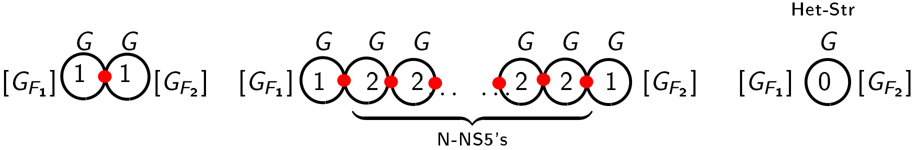

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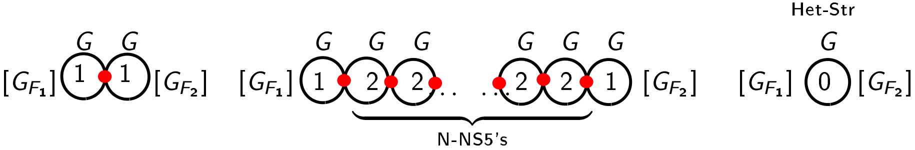

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Proof T-duality via Geometry

Proofing T-duality in F-theory

Construct **non-compact elliptic threefolds** over an LST base B_2:

$$\mathcal{E} \to X_3 \to B_2$$

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F/M-theory and T-duality

T-duality: Two theories are the same upon circle compactification:

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1. **Flavor groups** bound to be $G_F \in \Lambda_{K3} \in \text{Pic}_{K3}$
2. T-dual LSTs $\leftrightarrow$ elliptic fibrations in fixed K3 [Braun, Kimura, Watari'13/14]
3. **Gaugings** are Kulikov II/III degenerations[2] [Braun, Watari'16; Lee, Weigand'21]

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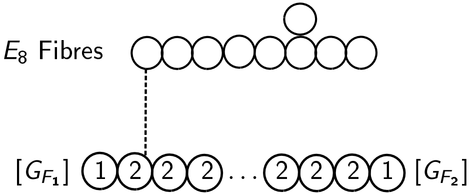
Fibre-Base-Duality

Construction of T-dual theories using **toric geometry and (semi-)convex polytopes** [Huang Taylor'18, Anderson, Gao, Gray, Lee'16]



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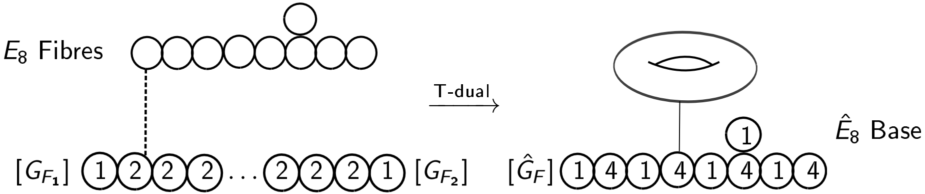

- **Affine Fibral** components become the T-dual Base
- T-dual **LST charges** $N^I = d^I/\delta^I$ Kac labels of gauge group G
- T-dual **base topology determined by gauging, not flavor groups** [3]

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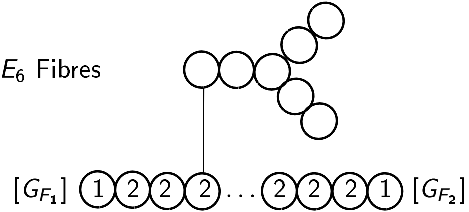
Fibre-Base-Duality

Fibre Base **duality holds for all gaugings** G

E_6 Fibres



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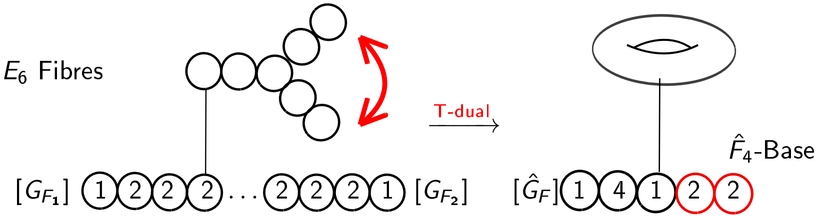

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- Base topology **reduced** by $\hat{G}$ **automorphisms** [Blum, Intrilligator '96]
- **No clear field theory reason why?**

Summary and Conclusion

Considered 6D (T-dual) heterotic Little String Theories

Summary

1. **Predict new T-dual pairs** by imposing match of **2-group symmetries**
2. **Proofen the duality** via geometric engineering
3. **Fractionalization** of single NS5 brane and **pure heterotic string** by G ADEs

Highlights

1. **Fibre base dualities**, sometimes **reduced** by $\hat{G}$ automorphism
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Outlook

- **Twisted T-dualities**
- **Non-heterotic LSTs** and their T-duals
- **Complete Classification**